Summary
A basic idea of a with profit policy is that the policyholder accepts a lower guarantee (sum assured) for the same premium, than he would accept under an equivalent without profit policy. In return for a lower guarantee, the policy has the right to share in the profits of the life insurance company. In order to qualify for such entitlement the policyholder will pay a higher premium that if the benefit was an amount stated in the contract. This paper describes a number of different methods that are used in various parts of the world to allocate profits to policyholders.

Key words
With profit contracts, reversionary bonuses, terminal bonus, contribution method, revalorisation method.

1. Introduction
Since 1994 the Department of Mathematics of the Faculty of Economic Informatics of the University of Economics in Bratislava has been involved in the teaching of actuarial science. A great help in this was the organising by the Department, in cooperation with the Know-How-Fund of the United Kingdom Government, two Postgraduate Diploma Courses in this field. These Courses were held from Spring 1994 until Summer 1996 and use was made of lecturers from the British Actuarial Profession.

Actuarial teaching is now organised as a second-level university course and there is also a possibility continuing to do PhD studies. Graduates from the course have been successful in finding employment as actuaries not just in insurance companies in Slovakia but also outside the country.

Actuarial science in Central Europe is most often made use of in the area of insurance, both life insurance and non-life insurance. People with actuarial knowledge prepare reports on the financial condition of the companies for the benefit of their managements and also for presentation to insurance supervisory authorities. In these reports they make use of their skills primarily to place a value on the future commitments, known as liabilities, of the companies. These liabilities, particularly in the case of life insurance, can stretch many years into the future and their amount depends on future developments in mortality and other decrement rates, investment returns and expense inflation ([6]).

This paper considers a particular type of life insurance contract, known as a with profit contract. The basic idea of a with profit contract is that the policyholder accepts a lower guarantee (basic sum assured) for the same premium, than he would accept under an equivalent without profit policy. In return for this lower guarantee, the policy has the right to share in the future profits made by the life insurance company. In order to qualify for such entitlement the policyholder will pay a higher premium that if the benefit was an amount stated in the contract. In the latter case the contract is known as a “without profits” contract.

A number of different methods are used in different parts of the world to allocate profits to policyholders. These methods have their advantages and disadvantages and have been developed to suit the environments in which they are used.

2. Additions to benefits
This method is used in the United Kingdom and many of the countries that are of the British Commonwealth, for example Australia, India and South Africa. The profit is distributed by making additions to the basic benefit, i.e. sum assured, under the contract ([3]). These additions are known as bonuses and take a number of different forms.

Reversionary bonuses
A reversionary bonus is an addition made to the benefit each year. Once a reversionary bonus has been added to the benefit, it cannot subsequently be removed. The amount of the bonus is not payable immediately it is added to the contract, but at the same time as the basic benefit is paid, i.e. on death or maturity.

The amount of the bonus can be calculated in one of three ways ([2]):

- simple - the bonus is expressed as a percentage $b$ of the sum assured $S$ under the contract. Thus after $t$ declarations the total guaranteed benefit, assuming that the rate $b$ applies for all $t$ years, is $S(1 + bt)$
compound - the bonus is expressed as a percentage \( b \) of the initial benefit \( S \) plus any bonuses that have already been given. Thus if again we assume that the rate stays at \( b \) we have after \( t \) declarations a total benefit of
\[
S(1 + bt)
\]

super-compound - the bonus is expressed in terms of two percentages \( b_1, b_2 \): one applied to the initial benefit and a second applied to any bonuses that have already been given. Thus we have as the total benefit after \( t \) declarations
\[
S(1 + b_1 + b_2 \frac{b_1}{1})
\]

The second percentage will usually be higher than the first.

If we have, for example a 25-year endowment contract with a sum assured of € 1 000. Suppose we want the reversionary bonus to amount to € 1 000 at maturity. The required simple rate is 4 %, compound rate is 2,81 %. For super compound we have more choices, but we could, for example take a rate of 2 % on the sum assured and 5,35 % on already given bonuses.

The main points to notice are these:

- The compound bonus builds up more slowly than the simple bonus, and the super compound bonus builds up more slowly than either.
- The cost of the simple bonus starts at a relatively high level, but is lowest by the end of the term.
- The cost of the compound bonus is lower than the cost of the simple bonus to start with, but higher at the end of the term.
- The super compound bonus is cheapest to start with, but very much the most expensive at the end of the term.

The major thing is that the bonus addition increases the guarantees. It becomes an additional promise to pay benefits. So the lower the bonus the lower the guarantees and a lower need to reserve for them.

**Terminal bonuses**

A terminal bonus is an addition to the benefit that is made when the benefit is actually paid and the amount of the addition will be determined at that point. This could in theory imply a constantly changing bonus, but in practice this does not happen, although a company will not usually guarantee to maintain the bonus at any particular level. The bonus to give to a particular contract may be specified in a number of different ways, for example

- a percentage, that may vary by duration in force, of the total reversionary bonuses that have already been added to the benefit;
- a percentage of the total claim amount, before addition of terminal bonus, with the percentage varying according to duration in force.

Current practice is the following: the company keeps the sum assured low to start with. It will keep the reversionary bonuses low, in order to reduce the rate at which the guarantees build up. Reducing the reversionary bonuses will reduce the cost of the bonus; so more surplus can be put into the investment reserve. This reserve can cushion the fluctuations of investing in riskier assets; investment freedom of the life insurance company is increased. At maturity, the sum assured plus bonuses will be lower than the policy’s asset share. In order to pay the maturity value, make a final transfer out of the investment reserve on the maturity date, and use this to pay a terminal bonus.

The key features of a terminal bonus, and the main differences between it and reversionary bonuses, are:

- Terminal bonuses are not guaranteed in advance.
- Terminal bonuses do not increase the guaranteed liabilities, and hence the need for valuation reserves, in the way that reversionary bonuses do.
- Terminal bonus is a retrospective payment, whereas a reversionary bonus is a prospective payment.

### 3. Contribution method

This method is used in North America and many of the countries on the Pacific Rim, for example Japan and South Korea.

Profit is given to a contract in the form of a dividend. The method is based on an analysis of the sources of a life insurance company’s profit and develops a dividend formula \([1]\). In the interest of simplicity consideration may be limited to the three major sources: excess interest, mortality savings and expense loading savings. Dividends are distributed annually.

Traditionally, the amount of dividend to be given to a particular contract was calculated using a formula such as

\[
dividend = I_r + M_r + E_r
\]
\[
\text{value of contract at beginning of year on valuation basis}
\]

\[
\text{value of contract at end of year on valuation basis}
\]

\[
\text{gross premium}
\]

\[
\text{actual rate of interest earned or so called dividend rate}
\]

\[
\text{valuation basis rate of interest}
\]

\[
\text{actual rate of mortality experienced}
\]

\[
\text{valuation basis rate of mortality}
\]

\[
\text{sum assured}
\]

\[
\text{actual expenses experienced under the contract}
\]

\[
\text{expenses experienced under the contract according to the valuation basis}
\]

The interest factor \( I \) is the simplest element but has a strong influence on the dividend particularly at long durations where the reserve is large. Hence the most important complication with this method is with the evaluation of the interest factor.

The dividend is often converted into an addition to the benefit, instead of being paid out in cash each year. Dividends can be also paid in a form of an extra dividend or terminal dividends in addition to the regular annual dividends. An extra dividend may consist of a single payment made after a policy has been in force a specified number of years. The single payment extra dividend is generally used when no first year dividend is paid, the extra dividend serving as a substitute. Some insurers pay also a terminal dividend.

Given now the ready availability of significant computer resources, a more complicated approach would usually be used, for example by taking into account more factors than does the simple formula above. Whichever method is used to calculate the dividend, a terminal dividend may also be given to reflect any profit, which has not yet been given to the policyholder.

4. Revalorisation method

This method and variations on it are used in Europe, for example in Germany, Slovakia ([4]), etc.

The profit given to a particular contract is expressed as a percentage of that contract’s supervisory reserve. The benefit under the contract and the premium payable by the policyholder are then increased by the same percentage. Typically, where this method is used the profit of the life insurance company is split into a “savings” profit and an “insurance” profit.

The “savings” profit is taken as that arising from the assets. For a particular contract, the savings profit at some time \( t \) can be expressed as

\[
V_t (i' - i)
\]

where

\[
i' = \text{actual rate of return on the assets}
\]

\[
i = \text{expected rate of return on the assets}
\]

\[
V_t = \text{reserve for the contract at time } t
\]

This is in a suitable form for distribution, in whole or in part, by the “revalorisation” method. The “insurance” profit is that arising from actual experience being better than expected for all sources of profit other than the return on the assets. This profit might then typically be retained by the company for distribution to shareholders, as reward for the pure insurance risks that they have borne.

5. Conclusion

As can be seen from the descriptions, the contribution and revalorisation methods are both based on a split of the actual profit arising each year. The main difference is that under the revalorisation method only the savings profit is usually distributed to policyholders, whereas with the contribution all sources are usually shared between the shareholders and the policyholders ([3]).

The additions to benefits starts from a quite different philosophy and its great virtue is that it can provide the insurance company with significant investment freedom which in turn should lead to better total benefits for policyholders.

References

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